

Orientifolds, Branes, and Duality of 4D Gauge Theories

Nick Evans^a, Clifford V. Johnson^b and Alfred D. Shapere^c

^a*Department of Physics, Boston University, Boston, MA 02215 USA*

^{b,c}*Department of Physics and Astronomy, University of Kentucky, Lexington, KY 40506 USA*

Abstract

Recently, a D-brane construction in type IIA string theory was shown to yield the electric/magnetic duality of four dimensional $\mathcal{N}=1$ supersymmetric $U(N_c)$ gauge theories with N_f flavours of quark. We present here an extension of that construction which yields the electric/magnetic duality for the $SO(N_c)$ and $USp(N_c)$ gauge theories with N_f quarks, by adding an orientifold plane which is consistent with the supersymmetry. Due to the intersection of the orientifold plane with the NS-NS fivebranes already present, new features arise which are crucial in determining the correct final structure of the dualities.

March 1997

email: ^anevens@physics.bu.edu, ^bcvj@pa.uky.edu, ^cshapere@pa.uky.edu

1. Introduction

1.1. Motivation

Recently, a qualitatively new approach to applying string theory to the study of field theory dualities has emerged. The field theories are realised as limits of string vacua which are constructed as configurations of intersecting (Ramond)² (R-R) branes and (Neveu-Schwarz)² (NS-NS) branes in flat ten dimensional spacetime. There is no involvement of non-trivial background fields representing a curved background compact geometry. The structure of the models is supplied purely by the intrinsic complexity of the brane configurations themselves.

In ref.[1], a type IIB string theory realisation of the $\mathcal{N}=4$ three dimensional ‘mirror’ dualities of ref.[2] was presented. It employed an intricate interplay between an NS-NS fivebrane[3], a D5-brane and a family of parallel D3-branes¹. The world volumes of the branes were all flat and fully extended, except for one dimension of the D3-branes, which was a finite interval whose length was set by the distance between the fivebranes. The $\mathcal{N}=4$ field theory was realised on the (infinite part of) the world volume of the D3-branes.

The mirror duality was implemented by an exchange of the two species of fivebrane (together with a rotation in some of the coordinates). A crucial ingredient was knowledge of the result of moving an NS-NS fivebrane past a D5-brane. It is not a straightforward matter to deduce the result of such a motion directly from string theory, as this requires more knowledge about the description of NS-NS string solitons than is presently available. This is largely because the string coupling diverges at their core, taking us out of the regime where we can presently directly calculate. However, continuity of the BPS spectrum led the authors of ref.[1] to realise that after moving an NS-NS fivebrane past a D5-brane a new D3-brane must appear stretched between them. This new feature was essential in reconstructing the final dual theory.

In four dimensions, there are a number of situations where the infrared (IR) limits of certain $\mathcal{N}=1$ supersymmetric gauge theories have dual descriptions. In the case[8] of gauge group $U(N_c)$ with N_f flavours of quark, (denoted Q^i, \tilde{Q}^j $i, j = 1, \dots, N_f$) in the fundamental representation (the ‘electric’ scenario), the IR limit of the theory has a dual description in terms of a $U(N_f - N_c)$ gauge theory with N_f quarks (q^i, \tilde{q}^j) in the fundamental (the ‘magnetic’ scenario), together with a N_f^2 -component gauge singlet meson, M_{ij} . There is also a coupling in the superpotential of the form $M_{ij}Q^i \cdot \tilde{Q}^j$.

For the case[9] of gauge group $SO(N_c)$ (or $USp(N_c)$) with N_f flavours of quark, there is also a dual description of the IR regime, this time in terms of an $SO(N_f - N_c + 4)$ (or

¹ We refer the reader to the literature for explanations of the term ‘D-brane’[4] and ‘orientifold’[5], (to appear later). For the definitive review (to date) see ref.[6]. See also ref.[7] for a fine review of other string soliton techniques.

$USp(N_f - N_c - 4)$ gauge theory with N_f flavours of quark. There is again a meson, a symmetric tensor under the global flavour symmetry for SO , (or an antisymmetric tensor for USp), and a superpotential $M_{ij}q^i\tilde{q}^j$.

In ref.[10], the duality for the $U(N_c)$ models was described using a type IIA string theory configuration inspired by ref.[1]. This time, the ingredients were two NS–NS fivebranes, arranged differently in the ten dimensional space, a family of N_c D4–branes, and a family of N_f D6–branes. One of the dimensions of the world volume of the D4–branes was a finite segment stretched between the fivebranes. The field theory of interest lives on the infinite part of the world volume of the D4–branes.

The important feature here was inherited from the discussion of ref.[1]: When NS–NS fivebranes move past D6–branes, there is a new D4–brane stretched between them. This conjectured behaviour was exploited later in the construction to obtain the correct dual theory. In addition, another type of unfamiliar strong coupling behaviour could potentially have arisen in that paper: The physics of two NS–NS fivebranes (with D4–branes connected to them) passing through one another. This type of situation was avoided by moving one fivebrane *around* the other, using the freedom to move in transverse directions.

In this paper we present a description of the SO/USp dualities in the spirit of refs.[1,10]². To do this we add a new ingredient, the orientifold. As is by now well known (see ref.[6] for a review), the orientifold is an extremely natural way of introducing orthogonal and symplectic gauge groups into type II string theory vacua, the type I string theory itself being the prototype example, with gauge group $SO(32)$.

Most simply put, the orientifolding procedure combines the gauging of fundamental string worldsheet parity Ω with target spacetime discrete symmetries, resulting in the introduction of non–orientable string sectors into the theory. The fixed points (in spacetime) of the discrete symmetries are called ‘orientifold planes’ (or sometimes just ‘orientifolds’) and can have any dimensionality. We shall call orientifolds with a $(p+1)$ dimensional fixed plane ‘ Op –planes’, in analogy with the term ‘ Dp –branes’ we have been using for a $(p+1)$ dimensional object.

There are a number of similarities and differences between O –planes and D –branes. They are both extended objects. They both typically break half of the supersymmetries. They both couple naturally to the R–R sector fields in the theory. However, while the D –branes are dynamical objects, the O –planes are not, at least in perturbation theory.

There has been much work over the last year in studying string vacua containing both

² Our approach is complementary to that of ref.[11]. There, a relationship between NS–NS fivebranes and certain singularities of Calabi–Yau manifolds is used to rephrase the results of refs.[1,10] in a geometrical context. That paper then presents a discussion of the introduction of an orientifold into the geometrical framework in order to derive the SO/USp dualities. (We thank H. Ooguri and C. Vafa for pointing out ref.[11] to us after the appearance of an earlier version of this manuscript.)

orientifolds and D-branes. (Indeed, in many of those cases, due to a compact transverse geometry the presence of one tends to demand the presence of the other in order to satisfy one-loop consistency.) There has been less work done in the context of mixing O-planes and NS-NS branes. This is largely because of the lack of a complete description of the latter type of object, a situation not unrelated to the aforementioned strong coupling region at the core.

So although it is natural to introduce O-planes to facilitate the description of the SO and USp dualities, we will inevitably have to consider new phenomena. On one hand, this is fortunate, as without new phenomena we will not be able to describe the details of the duality correctly. (For example, there will be a necessity for a pair of D4-branes to appear (or disappear) as we move between dual descriptions. Furthermore, at least for $SO(N_c)$ gauge theories, there is a genuine phase transition, not present for the $SU(N_c)$ theories[8][12], which must also occur.) On the other hand, we shall have to make some guesses about new phenomena in strongly coupled string theory, as the orientifold forces us to consider new strong coupling regions of the scenarios we construct. On a third hand, we can find strong justification of our new stringy results by appealing to certain known properties of $\mathcal{N}=2$ theories which are highly suggestive of non-perturbative physics attributable to M-theory. So given that these new phenomena yield the $\mathcal{N}=1$ field theory duality results we are studying, we can be very satisfied that this enterprise has taught us some new details about a thorny problem in strongly coupled string theory.

In the remainder of this introduction we recall some of the details of the construction used in ref.[10] to realize electric and magnetic $U(N_c)$ theories and the duality between them, within type IIA string theory. In section 2, we show how to do the same for $SO(N_c)$ and $USp(N_c)$ theories by introducing an orientifold. Here it will be necessary to make an assumption about the behaviour of orientifolded NS-NS fivebranes at strong coupling. Evidence for our assumption will be found in section 3, where we present a similar brane realization of $\mathcal{N}=2$ gauge theories. Specifically, we will find that the auxiliary Riemann surface of Seiberg and Witten[13] arises naturally, allowing us to directly identify the moduli of the brane configurations with parameters of the effective field theory. This correspondence is interesting in its own right, and has been recently independently studied by Witten [14] in the context of $U(N_c)$ gauge theories. While our paper represents work that was completed before ref.[14] appeared, we expect that Witten's work will play an important role in future developments.

In Section 4, we give a generalization of the construction of Section 2, which may describe theories with adjoint matter. Finally, we summarize our conclusions in Section 5.

1.2. The ‘electric’ $U(N_c)$ gauge theory.

The brane configuration of ref.[10] may be summarized by the following table³:

type	#	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
NS	1	—	—	—	—	—	—	•	•	•	•
NS′	1	—	—	—	—	•	•	•	•	—	—
D4	N_c	—	—	—	—	•	•	[—]	•	•	•
D6	N_f	—	—	—	—	•	•	•	—	—	—

Table 1.

In the table, a dash ‘—’ represents a direction *along* a brane’s worldvolume while a dot ‘•’ is transverse. For the special case of the D4–branes’ x^6 direction, where a worldvolume is a finite interval, we use the symbol ‘[—]’. (It is particularly simple to read off a lot of information from such a table. For example a ‘•’ and a ‘—’ in the same column says that one object is living inside the worldvolume of the other in that direction, and so they can’t avoid one another. Meanwhile two ‘•’s in the same column tell us that the objects are pointlike, and need not coincide in that direction, except for the specific case where they share identical values of that coordinate.)

The N_c coincident D4–branes give rise to a $U(N_c)$ gauge symmetry on their worldvolumes. This symmetry arises from massless fundamental strings (‘4–4 strings’) connecting the various branes, in the usual way. Focusing on the four dimensions of the (x^0, x^1, x^2, x^3) directions, we have a gauge theory with coupling strength given by $g^2 \sim 1/L_6$, where L_6 is the distance the D4–branes are stretched between the two fivebranes in the x^6 direction.

The N_f (not necessarily coincident) D6–branes contribute (via 6–4 strings) matter fields to the $U(N_c)$ gauge theory, transforming in the N_c dimensional fundamental representation. There are N_f flavours of such quarks.

There are also 6–6 strings, whose role is to supply a flavour symmetry to the problem which is generically $U(1)^{N_f}$, but can be as large as $U(N_f)$. As this symmetry is a gauge symmetry on the seven dimensional world volumes of the D6–branes, it is best thought of as a global ‘spectator’ symmetry from the point of view of the dynamics of the D4–branes’ worldvolume gauge theory. It will not play a major role in the proceedings.

³ The temptation to term such a table a ‘brane–scan’ is nearly overwhelming.

That it is $\mathcal{N}=1$ supersymmetry which is present in four dimensions follows from analyzing the conditions on the spinor generators imposed by the worldvolumes of the various objects. The analysis is already presented in ref.[10], generalising the presentation in ref.[1], and will not undergo any modification here. The configuration preserves 1/16 of the original ten dimensional $\mathcal{N}=2$ supersymmetry.

For definiteness, take the configuration giving this ‘electric’ type description of the field theory to have the first NS–NS fivebrane (denoted NS), to the left of the second NS–NS fivebrane (denoted NS′) in the x^6 direction. There are N_c D4–branes stretched between them, along that direction, passing N_f D6–branes along the way. (See Fig. 1.⁴)

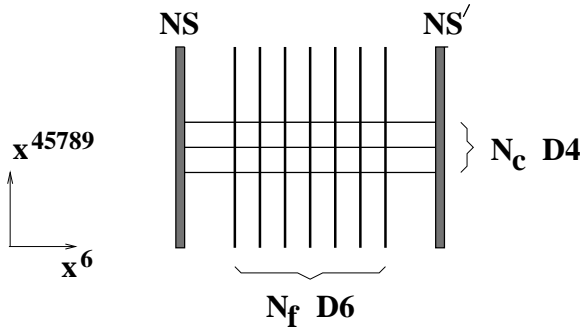


Figure 1.

Note that the translational degrees of freedom of the string model have a one-to-one mapping to moduli of the gauge theory. The distances between the D6–branes and the D4–branes in the (x^4, x^5) direction correspond to mass terms for the matter fields. Segments of D4–branes connecting to the NS′ fivebrane and touching a D6–brane, and those stretching between D6–branes, are free to move in the (x^8, x^9) direction. In general, such movement will leave the N_c D4–branes non-coincident, and therefore these directions correspond to vacuum expectation values (vev’s) for the matter fields that break the gauge symmetry.

The final translation that may be made is to move the two NS–NS fivebranes relative to each other in the x^7 direction. This may be done if we also shift the x^7 positions where the N_c D4–branes touch the D6–branes. This potentially breaks supersymmetry by introducing a Fayet–Iliopoulos (FI) term into the potential of the field theory. With massless matter fields present, supersymmetry breaking may be avoided by turning on matter field vev’s that generate a non-zero D –term (canceling the FI term) and break the

⁴ Note that in this and all other figures, we have displaced the D4–branes away from coincidence, to aid with visualisation, and we have ignored the (x^0, x^1, x^2, x^3) directions which are common to all of the branes. Also, we will only indicate on a diagram whether the branes are pointlike or extended in the $(x^4, x^5, x^7, x^8, x^9)$ directions when necessary. That information may be found in Tables 1 and 2.

gauge symmetry. Such FI terms can only arise in theories which have a $U(1)$ centre of the gauge group⁵.

1.3. The ‘magnetic’ $U(N_f - N_c)$ gauge theory.

Continuing to follow ref.[10], the dual description of this theory is obtained by exchanging the positions of the fivebranes in the x^6 direction. In order for this to happen, the NS fivebrane has to first move past the D6-branes, which are at a definite values of x^6 .

This is where the observation of ref.[1] is crucial. A study of the spectrum in the world-volume gauge theory of a related situation (NS–NS fivebranes with D3-branes stretched between them, passing through D5-branes) showed that there must be a new stretched brane between the NS–NS fivebrane and the D-brane it passed through. This may be deduced by insisting that if the movement is a true modulus of the theory (which it is, as we can see in the field theory since changing the ultra-violet (UV) coupling does not effect the far IR behaviour of the theory) then the BPS spectrum must be the same before and after the encounter. In order that there be the same hypermultiplet structure before and after, the most conservative explanation is that there is a new stretched D3-brane (the new hypermultiplet corresponds to strings connecting the new D3-brane and the old D3-brane). This surmounts the problem of trying to describe directly the strong coupling string physics lurking at the core of the fivebrane due to the growth of the dilaton there.

So in the present context, when the NS fivebrane has moved past all of the D6-branes, there are N_f new D4-branes stretched in the x^6 direction. In particular, there is one stretching from each of the N_f D6-branes to the NS fivebrane. There are still N_c D4-branes between the two fivebranes. (See Fig. 2.)

The next step is to move past the NS’ fivebrane. In moving the NS fivebrane — with its entourage of D4-branes — through the NS’ fivebrane, there is the aforementioned problem that we have little knowledge of the description of string theory in such a situation. The presentations in refs.[1,10] cunningly avoided this problem by going *around* the potentially singular behaviour.

⁵ It is easy to see on general grounds that this coupling must enter the field theory in this way. So far, everything else in the effective field theory are fields from the open string sector, coming in vector- and hyper-multiplets of the gauge and supersymmetry. The positions of the NS–NS fivebranes are governed by the closed string sector, contributing a new type of term to the Lagrangian which does not transform under the gauge symmetry of the open string sector. It enters as a gauge invariant supersymmetry-breaking term. The possibility of restoring supersymmetry by a Higgs mechanism allows for a new direction in the moduli space of vacua, a Higgs phase. Similiar reasoning has been used in other situations, for example in identifying the FI terms corresponding to blowing up an ALE space[15,16,17].

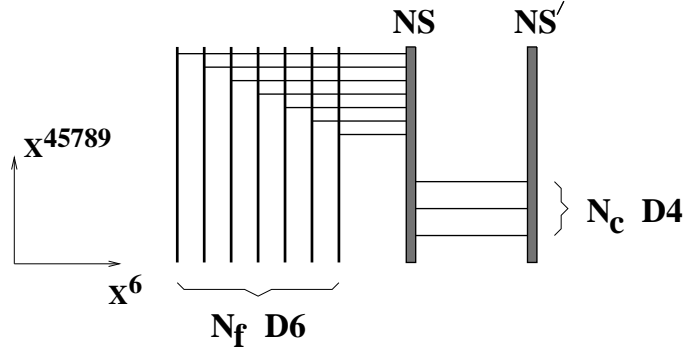


Figure 2.

There is the possibility to move the NS fivebrane off to a different x^7 value than where the NS' fivebrane is located. It can then go around and return to its original x^7 value once it has moved far enough in the x^6 direction, thus ending up to the right of the NS' fivebrane, achieving the desired final configuration without encountering a new region of strong coupling.

As a result of the NS fivebrane moving off into the x^7 direction, the N_c connecting D4-branes can no longer stretch directly between it and the NS' brane, as an examination of the configuration table confirms. Instead, they connect from the NS' brane to N_c of the N_f D6-branes (which are sharing the (x^4, x^5) position of the NS' brane). The remaining $N_f - N_c$ D6-branes retain their connection to the NS brane. (See Fig. 3.)

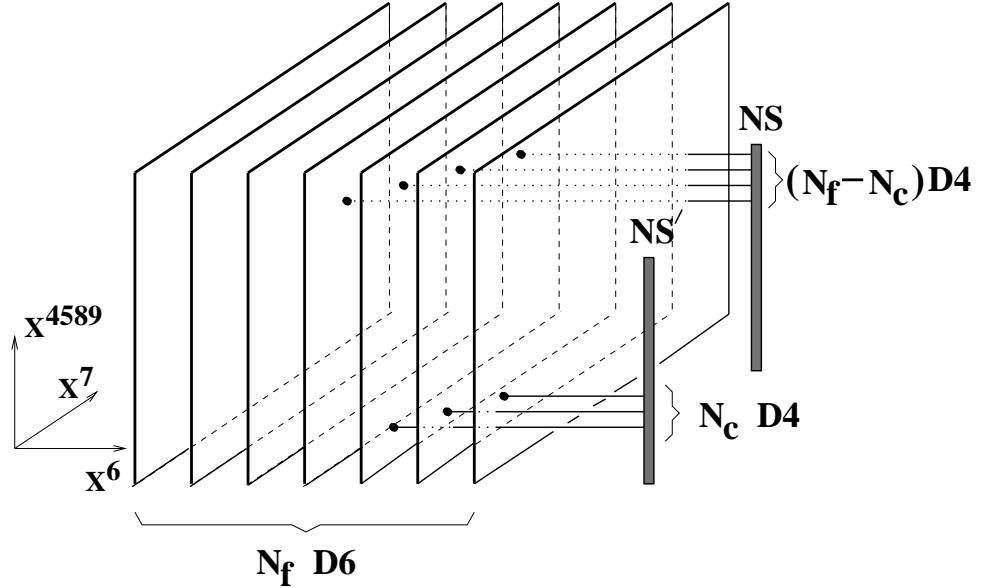


Figure 3.

The field theory is now in a Higgs phase. The distance between the x^7 positions of the

two fivebranes corresponds to an FI term in the scalar potential of the theory. In order to ensure supersymmetry, a new zero of the scalar potential may be found by breaking the gauge group with a Higgs mechanism, which is achieved by the just-described movement of the D4-branes.

After moving around the NS' brane, the NS brane may return to its original position in x^7 . The FI term disappears and a gauge symmetry returns with the possibility of reconnecting the fivebranes directly with D4-branes. The $N_f - N_c$ such branes connecting the NS brane to the $N_f - N_c$ D6-branes now split, reconnecting free ends to the NS' brane. The N_c D4-branes connecting the D6-branes to the NS' brane are now accompanied by the $N_f - N_c$ D4-branes from the other half of the split, now making the same D6-NS' connection. (See Fig. 4.)

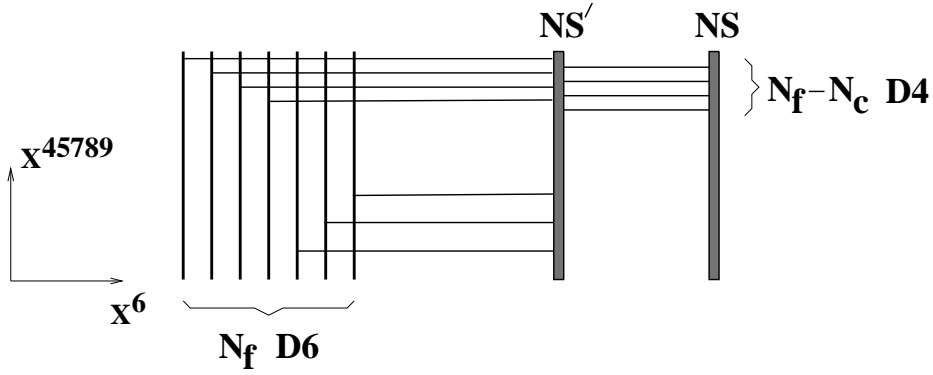


Figure 4.

The worldvolume theory of the D4-branes is now as follows. There is a $U(N_f - N_c)$ gauge theory (from 4-4 strings between the fivebranes) with N_f flavours (from 4-4 strings across the NS' fivebrane) of quark, q_i , in the fundamental. There is also a family of N_f^2 fields coming from the fluctuations (in (x^8, x^9)) of 4-4 strings connecting the N_f D4-branes. This is the meson field M_{ij} . Its coupling to the quarks in the superpotential may be deduced by examining the last figure and considering how to turn 4-4 strings which define the quarks into 4-4 strings which make the meson.

This is the 'magnetic' dual description of the original $U(N_c)$ field theory, as first presented in ref.[8]. The 'loom' arrangement above, weaving D4-branes between fivebranes and D6-branes, bears fruit by making this $\mathcal{N}=1$ duality manifest, an (almost) simple consequence of being embedded in string theory in this way.

2. Electric/magnetic duality in $SO(N_c)$ and $USp(N_c)$ gauge theories.

In this section we describe how to construct the electric/magnetic duality for the $SO(N_c)$ and $USp(N_c)$ gauge theories with N_f quarks. On general grounds, we expect that this will

not be as straightforward as the construction for the $U(N_c)$ gauge theories (with N_f quarks) for the following reason: There is no true phase transition in going between the electric and magnetic descriptions for the $SU(N_c)$ case, while there is a phase transition for $SO(N_c)$ [12]. The occurrence of a phase transition is detected with an order parameter, which in this case is a Wilson loop (in any representation). In the case of $SU(N_c)$ with matter in the fundamental, any phase transition which could be detected by such an order parameter is ‘screened’ by the quarks. This is because any representation can be constructed with the fundamental (and anti-fundamental) representations. In the case of $SO(N_c)$, matter in the fundamental cannot screen sources which are in spinor representations (which cannot be made out fundamentals) and so there is a viable order parameter. A phase transition in going between the electric and magnetic descriptions can be detected by observing that a Wilson loop expectation value’s dependence on the geometry changes from an area law to a perimeter law, or *vice versa*.

We do not expect, therefore, that we should be able to find a description of the path between the magnetic and electric variables which is as smooth as the one found in ref.[10]. We do expect that at some point on the path between the two phases, a non-trivial point *must* be encountered which allows for the occurrence of a phase transition. Indeed, we will find in this section that such a point is *forced upon us* by the presence of the orientifold which we introduce in order to construct the $SO/USp(N_c)$ cases.

2.1. The new ingredient: An orientifold.

Orthogonal and symplectic gauge groups arise naturally in string theory in the presence of D-branes by orientifolding.

In general, the procedure of orientifolding will reduce the amount of supersymmetry by half. This happens in much the same way as it happens for D-branes. The worldvolume of an O-plane reflects the supersymmetry generators, leaving only a linear combination of the ingoing and outgoing spinor to carry the remaining supersymmetry. In general, that will spoil our present arrangement considerably, as we will fully break the four dimensional supersymmetry. It is possible, however, in this situation to perform an orientifold in such a way as to preserve the supersymmetry that is already present.

We need only orientifold in such a way as to create an O-plane whose world-volume lies in the same dimensions as are already occupied by the worldvolume of a D-brane. Then the O-plane’s world volume will place conditions on the spinors which are already satisfied, thus preserving the supersymmetry of our arrangement.

After a little thought, it is obvious that we need only add an O4-plane to our arrangement to get the desired result⁶. This O4-plane must extend in the x^0, x^1, x^2, x^3, x^6 directions to

⁶ Of course, we can also add an O6-plane parallel to the D6-branes and preserve the same amount of supersymmetry. We will not do this here.

preserve supersymmetry. Such a plane results from combining the gauging of world sheet parity Ω with the spacetime reflection

$$(x^4, x^5, x^7, x^8, x^9) \rightarrow (-x^4, -x^5, -x^7, -x^8, -x^9).$$

(Of course, if these directions were compact, the resulting orbifolded torus would have 32 O4-planes, which is rather more than we need.)

Note that the O4-plane is not of finite extent in the x^6 direction. Attaching it to the NS and NS' fivebranes and moving it around with them would be tantamount to making it dynamical, which it is not, at least in string perturbation theory.

2.2. The ‘electric’ $SO/USp(N_c)$ gauge theories.

The configuration table for our new electric scenario is as follows:

type	#	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
NS	$\frac{1}{2}$	—	—	—	—	—	—	•	•	•	•
NS'	$\frac{1}{2}$	—	—	—	—	•	•	•	•	—	—
O4	1	—	—	—	—	•	•	—	•	•	•
D4	$\frac{N_c}{2}$	—	—	—	—	•	•	[—]	•	•	•
D6	$\frac{N_f}{2}$	—	—	—	—	•	•	•	—	—	—

Table 2.

Some of the basic effects of the orientifold are easy to describe, referring to the table. Consider the directions x^m where the orientifold plane is located at a point. Any object which is not coincident with it in those dimensions (say at $x^m=x_0^m$) will have a mirror copy of itself at $x^m=-x_0^m$. This is why we have the factors of one half in the counting of the number of physical objects in each row of the table. It would be overcounting to consider an object and its reflection as separate physical objects.

We will take $N_c/2$ D4-branes with their duplicates. Generically, the gauge group is then $U(1)^{N_c/2}$. If they are all coincident, it is $U(N_c/2)$. However, when they are all coincident and lying precisely on the O4-plane, strings between the $N_c/2$ D4-branes and their copies fill out gauge group $SO(N_c)$ or $USp(N_c)$. Whether the gauge group is $SO(N_c)$ or $USp(N_c)$ results from the choice of whether Ω^2 acts as ± 1 on the open string sectors[18].

Also correlated with the sign of Ω^2 is the $H^{(6)}=dA^{(5)}$ R-R charge of the orientifold plane. In the natural normalisation where the D4-branes carry one unit of this charge, the O4-plane carries ∓ 1 units, for $\Omega^2 = \pm 1$ in the D4-brane sector.

(It is worth noting that odd N_c is achievable by the introduction of “half-D4-branes” that are forced to remain in the O4-plane. In general though, without a more complicated scenario than we have here, only even numbers of half branes can move off the orientifold plane. This translates into a pattern of Higgsing (and giving mass terms in the dual theory) which can only change N_c by two. We can thus only deform the theory by relating even N_c theories or odd N_c theories, which is a subset of the possible deformations of the theory. For definiteness, we consider only even N_c theories, but note that we can consider odd N_c theories, with the mentioned restrictions on the type of deformations we can do. It is possible that there are other scenarios, which will yield the even-to-odd N_c deformations that we don’t see here.)

For the branes which are not completely parallel to the O4-plane, things are interesting. The O4-plane cuts through them, and reflects the physics on one side of the bisection into that on the other side. Differently put, the O4-plane places a reflecting boundary in the (parts of) the worldvolumes of the branes it intersects. Referring to the table, this happens for directions in which an object has a ‘—’ where the O4-plane has a ‘•’.

For the D6-branes, this is an interesting but completely innocuous situation from the point of view of computing in weak coupling string theory, as the orientifold is simply an additional projection condition over and above the Dirichlet boundary conditions which describe the D6-brane. If the D6-branes are moved off the O4-plane, multiple copies will be generated and it is then clear that $N_f/2$ D6-branes give rise to N_f matter multiplets⁷. Again odd numbers of flavours may be generated by the inclusion of half D6-branes fixed on the O4-plane.

However, for the NS–NS fivebranes, the physics of orientifolding is not as clear. There is certainly a partial description of this situation in terms of an orientifold of the conformal field theory[20] of (part of) these objects. Indeed, such a description is almost certainly related to some of the earliest non-trivial orientifolds, studied in the context of black hole physics in ref.[21]. There, the action of Ω was gauged in combination with a target space symmetry of a gauged WZW model. Notice that the ‘throat’ conformal field theories of NS–NS fivebranes are realised as closely related gauged WZW models. It would certainly be interesting to compute some of the details of such a new situation as an NS–NS fivebrane

⁷ Consistency of the string theory requires that the possible gauged flavour symmetry group coming from the $N_f/2$ D6-branes be $USp(N_f)$ or $SO(N_f)$ for the choices $\Omega^2 = \pm 1$ in the D4-brane sector. This is because Ω^2 will act with the opposite sign in the D6-brane sector, a fact that is T_{45789} -dual to the situation with D5- and D9-branes in type I string theory[18]. This requirement on the possible gauged flavour symmetry is known independently from a field theory perspective[19].

straddling an orientifold plane. (This opens up a potentially vast area of investigation: revisiting many non-trivial conformal field theories representing type II backgrounds and orientifolding them. However, we will leave that as a future direction of research, and press on with the errand of this paper.)

Considering an orientifolded NS–NS fivebrane in isolation for a moment, we can anticipate some of the principal players in the content of the resulting model. There will be a new family of closed string fields arising from orientifolding the closed strings making up the NS–NS fivebrane. These come from the twisted sectors of the orbifold part of the spectrum. In general, we expect that the O4–plane and the NS–NS fivebrane must carry ‘twisted sector charges’ under these closed string fields. As twisted sector fields have no zero mode (and are therefore localised), sources for them must remain trapped at the orbifold fixed point[22]. This must mean that a NS–NS fivebrane must remain on the O4–plane⁸. In this sense, the NS–NS fivebranes are really half–fivebranes, analogous to the half D–branes mentioned earlier.

Another way to see that the half–fivebrane is trapped on the orientifold is from the content of the field theory. The gauge group (either $SO(N_c)$ or $USp(N_c)$) now has no $U(1)$ center. Therefore any coupling arising in the theory corresponding to moving the half–fivebrane off into the x^7 direction cannot enter as a Fayet–Iliopoulos term, and therefore it is impossible to arrange to Higgs the gauge group in such a way as to make that type of movement a supersymmetric flat direction of the theory.

This would seem to place a spanner in the works of our weaving arrangement. Now, we cannot move the NS half–fivebrane off the orientifold in the x^7 direction, as we wish to do to mimic the constructions of refs.[1,10].

We therefore cannot avoid the strong coupling singularity of moving the NS–NS fivebranes through each other. Recall that we anticipated the necessity of encountering such a singular situation at the beginning of this section. Without such a new feature, it is difficult to see how the phase transition in moving between the magnetic and electric phases of the SO theories could occur. Given that this is so, we are still confronted with the fact that we have no accurate string theory description of what occurs at such a point. Undaunted, let us proceed to try to reach the magnetic theories.

2.3. The ‘magnetic’ $SO/USp(N_f - N_c \pm 4)$ gauge theories.

We begin with $N_c/2$ D4–branes stretched between the NS and the NS’ brane in the x^6 direction. There are also $N_f/2$ D6–branes between the two half–fivebranes, along the x^6 direction. Moving the NS half–fivebrane through the $N_f/2$ D6–branes generates $N_f/2$ additional D4–brane connections between the D6–branes and the NS half–fivebrane. Of

⁸ See refs.[23,16,17] for other situations of exactly this type.

course, we would expect this to be still true, as locally nothing significantly new is happening which would lead one to expect that new phenomena are occurring to change the massless spectra. We therefore can draw a figure much like Fig. 2, the only modification being the addition of the mirror plane. (See Fig. 5.)

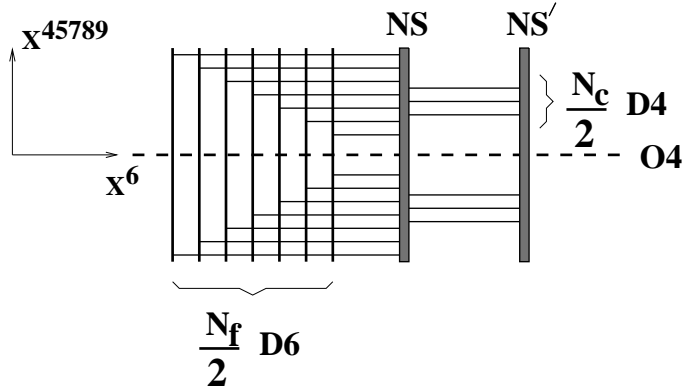


Figure 5.

The massless spectrum is now realised entirely by 4–4 strings, either stretching amongst the $N_c/2$ D4-branes to give the gauge sector, between the $N_f/2$ and the $N_c/2$ D4-branes to give N_f massless quarks, or amongst the $N_f/2$ to give flavour symmetry.

It is interesting to note at this stage that something non-trivial has happened to the O4-plane. All of the possible non-Abelian symmetries — both gauge and flavour — are carried by D4-branes, in contrast to the earlier situation where the flavour sector was carried by D6-branes. We have been careful to ensure that we have done nothing to the spectrum, so this is the same model as we had before moving the half-fivebrane past the D6-branes. However, we noted previously that the D6-branes carried a $USp(N_f)$ symmetry whenever the D4-branes carried an $SO(N_c)$ symmetry and *vice versa*. So in this new situation, it must be that the D4-branes to the *left* of the NS half-fivebrane carry the same non-Abelian symmetry as the D6-branes. Recall that the difference between the SO or USp choice was correlated with the sign of Ω^2 . Recall also that the sign of the $A^{(5)}$ charge of the orientifold was correlated with the sign of Ω^2 . Upon examination of Fig. 5., we are therefore able to conclude that, moving along x^6 , the sign of the $A^{(5)}$ charge of the O4-plane flips as it passes a half-fivebrane (NS) and then (by symmetry) flips back again as it passes the NS' brane. Along the x^6 directions, the half-fivebranes act as ‘domain walls’ with respect to the orientifold charge. They themselves have opposite twisted sector charges. We will use these observations to our advantage as we proceed.

Ultimately, we are going to have to approach the strong coupling singularity where we move the half-fivebranes to the same x^6 positions. Notice that this is strong coupling for both the field theory (whose coupling goes inversely with their x^6 separation) and for the string theory (because the dilaton blows up at the fivebrane cores), as it should be. This

is the only place where a new phenomenon can occur, and it happens just at the point where our ignorance about how to compute is greatest.

Let us assume for a moment that we have passed the NS half-fivebrane through the NS' brane successfully, passing to the other side, and recovered a candidate for the 'magnetic' dual theory. Let us see what we can say about this new configuration. Taking what we have learned from the $U(N_c)$ situation with N_f flavours, our first guess might be that perhaps there are now $(N_f - N_c)/2$ D4-branes between the half-fivebranes by analogy. Indeed this was justified in that case by passing one brane around the other.

After the fact, one can see that there is another argument for that resulting $N_f - N_c$ situation, based upon the fact that it is the only assignment of connecting D4-branes which preserves the local 'linking number' assignments to the fivebranes, following the arguments of ref.[1]. The linking number between two branes is a topological invariant calculated by integrating one brane's potential (for which it is a source) over the worldvolume of the other brane. One must also take into account the presence of the endpoints of other branes which end on the world volumes of the two branes in question, because the endpoints act as sources in the worldvolume theories. In the case of the $U(N_c)$ situation, the fivebranes have simply exchanged their positions on the x^6 line, producing no change in the contribution to linking number which involves their properties as sources, as they are identical. There is no option in preserving the total linking number but to redistribute the endpoint sources by reconnecting with $N_f - N_c$ D4-branes between the fivebranes.

It is not clear whether such a linking number assignment is able to restrict the physics in this case. We cannot completely compute the linking number in this situation as there is not enough knowledge about the detailed twisted sector couplings of the half-fivebranes and the orientifold. What we do know is that passing one half-fivebrane through the other is *not* completely analogous to the situation reviewed above, involving whole fivebranes. Due to the subtlety we noticed earlier concerning their role in flipping the sign of the orientifold's charge as one moves along the x^6 direction, we know that these are *not* identical objects under exchange. They carry opposite amounts of twisted sector charge.

We therefore conclude that we will not simply get $(N_f - N_c)/2$ D4-branes, which would be the case if we had passed identical objects through each other, but $(N_f - N_c)/2 + \alpha$, where α is to be determined. The value (including the sign) of α is ultimately computable with more knowledge about the twisted sector charges, which we do not have.

The result $\alpha = \pm 2$ (for $SO(N_c)$ and $USp(N_c)$, respectively) suggests itself, by comparison to the magnetic theory we are trying to recover. We cannot independently justify it at this stage of the discussion because we have no way of doing a strong coupling calculation. In the next section we will justify the claim that $\alpha = \pm 2$.

Assuming the result $\alpha = \pm 2$ for now, the final situation is thus as follows: After moving the NS half-fivebrane from the left, through the NS' brane to the right, we have a net number of $(N_f - N_c)/2 \pm 2$ D4-branes stretched between the two half-fivebranes which will

contribute to the massless spectrum. We have $N_f/2$ D6-branes to the far left, with one D4-brane each stretched between them and the NS' brane. (See Fig. 6.)

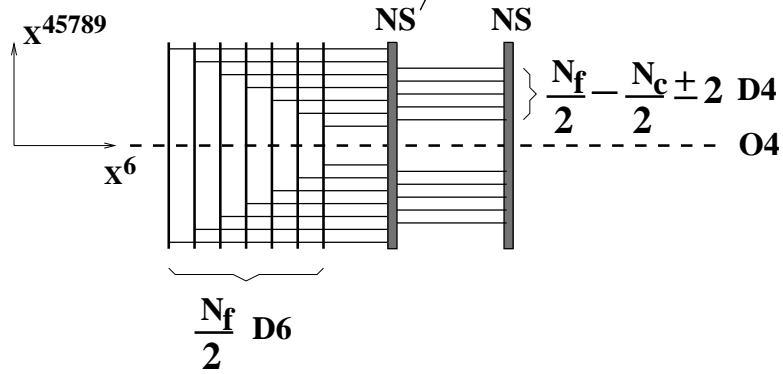


Figure 6.

This gives an $SO/USp(N_f - N_c \pm 4)$ gauge group coming from the 4-4 strings between the fivebranes, with N_f quark flavours in the fundamental (coming from the 4-4 strings connecting the two different D4-brane families). There is a meson associated to the (x^8, x^9) fluctuations of the $N_f/2$ D4-branes. As the D4-branes are precisely parallel to the O4-plane, the meson is the (anti) symmetric part of M_{ij} with couplings to the quarks in the superpotential, as before.

So we see that our orientifolded weaving arrangement has reproduced the dualities of refs.[8,9]. In order to recover this positive result, we had to guess that there was a discrepancy of ± 2 physical D4-branes after we passed the half-fivebranes through each other, over and above the appearance of $(N_f - N_c)/2$ one might guess from trying to generalise ref.[10]. That there is a different number than $(N_f - N_c)/2$ was a justified assumption, due to the presence of the orientifold. However, fixing it to $(N_f - N_c)/2 \pm 2$ needs independent strong coupling information about the string theory, which we discover in the next section.

3. Strong coupling and $\mathcal{N}=2$ physics.

Our goal is to find some independent means of deducing that there are precisely *two* new D4-branes which must appear (or disappear) when the NS-NS half-fivebranes pass through each other from the electric to the magnetic theory (or *vice versa*), in the presence of the orientifold. We need some sort of clue about where these extra branes might come from, and why we did not see them in the weak coupling theory (at least in the massless spectrum).

The clue appears when we deform our brane configuration to one in which the four-dimensional field theory has $\mathcal{N}=2$ supersymmetry. Consider for a moment the same scenario which we had before, but with the NS' half-fivebrane extended in the (x^4, x^5) directions and pointlike in the (x^8, x^9) directions, *i.e.*, parallel to the NS half-fivebrane. With the two half-fivebranes parallel, there are now twice as many supersymmetries as in the previous situation, and we have $\mathcal{N}=2$ supersymmetry in our field theory. (As recently described in ref.[24], such a configuration is continuously connected to the original situation by a rotation of the NS' brane.)

Notice, however, that it is still impossible to move the half branes around one another. (It is worth noting that now that the half-fivebranes are oriented the same way, they can move together, cancel their opposite twisted sector charges, and move off the orientifold, together with their mirror partner moving the opposite way. This is a degree of freedom which was not available in the $\mathcal{N}=1$ situation.) We still have an unavoidable strong coupling singularity. There is some hope, though, that the exactly solved $\mathcal{N}=2$ field theory might help us to understand aspects of the strongly coupled dynamics of these branes — specifically, the effect of passing two NS half-fivebranes through each other⁹.

Let us focus on the case of $SO(N_c)$ with N_f flavours and N_c even. If we study our $\mathcal{N}=2$ theory on the Coulomb branch, we have gauge symmetry $U(1)^{N_c/2}$, with N_f charged hypermultiplet fields. In this phase of the theory, much is known about its exact structure[13]. In particular, the Coulomb branch of the theory is controlled by the properties of an associated Riemann surface [26]:

$$y^2 = x \left(\prod_{a=1}^{N_c/2} (x - \phi_a^2)^2 + x^2 \prod_{i=1}^{N_f} (x - m_i^2) \right). \quad (3.1)$$

Here, m_i are the hypermultiplet mass parameters, ϕ_a are the vev's of photons in the theory with generic gauge group $U(1)^{N_c/2}$, and we have set the QCD scale Λ equal to 1. This equation describes a genus $N_c/2$ Riemann surface Σ as a double sheeted plane. The space of possible vacua of the Coulomb phase is the moduli space of such curves, and the prepotential for the low-energy field theory is determined by the periods and residues of an associated one-form λ on Σ [13]:

$$\lambda = \frac{\sqrt{x}}{2\pi i} d \log \left(\frac{x \prod (x - \phi_a^2) - \sqrt{xy}}{x \prod (x - \phi_a^2) + \sqrt{xy}} \right) \quad (3.2)$$

The residues of λ are linear combinations of the quark masses m_i , and are located at the points $x = m_i$. It is important to note that there is no residue or monodromy around

⁹ Dualities of $\mathcal{N}=2$ field theories have previously been used to understand $\mathcal{N}=1$ dualities in [25] and [19].

the point $x=0$, where there appears to be interesting behaviour due to the x^4 term in the curve [27]. This point will be of great interest to us below.

In the $\mathcal{N}=2$ theory, realised by D-branes, we can move through the Coulomb branch by simply moving the D4-branes around, giving them arbitrary and independent positions on the (x^4, x^5) plane. The strings which were connected to coincident D4-branes to give the non-Abelian gauge symmetry are now massive, their masses being proportional to the distances (in the (x^4, x^5) directions) between the various branes.

We propose that an identification should be made between the abstract cut plane describing the $\mathcal{N}=2$ field theory's vacua in the Coulomb phase and the (x^4, x^5) part of the world-volume of the NS-NS half-fivebranes, where the D4-branes end. Indeed, there is a one-to-one correspondence between the masses and vevs parameterizing the Coulomb branch and the D4-brane positions.

The precise correspondence is most easily made using a different parameterisation of the curve (3.1). Substituting $x=z^2$ and $y=zw$ (a generalization of the isogeny transformation of [28] and [26]) gives the genus N_c curve

$$w^2 = \prod_{a=1}^{N_c/2} (z^2 - \phi_a^2)^2 + z^4 \prod_{i=1}^{N_f} (z^2 - m_i^2). \quad (3.3)$$

This curve $\tilde{\Sigma}$ is a double cover of (3.1), with the projection identifying z with $-z$. The previous solution is obtained after modding out by this identification, leaving a set of periods and one-forms in one-to-one correspondence with those of the curve (3.1) [28].

In the new parameterization, we expect that an identification of the form $z=x^4+ix^5$ may be made. Then the above curve (3.3) should be viewed as being embedded in the covering space of the orientifold. The two sheets of $\tilde{\Sigma}$ are to be identified with the (x^4+ix^5) -planes of the two half-fivebranes. Punctures of the curve are to be identified with D4-branes ending on the NS half-fivebranes at the corresponding locations. Reading off from the curve and its associated one-form, we see that for every D4-brane ending at a position $z=m_i$, there is another brane at $z=-m_i$. In the weak-coupling limit of small Λ , there are also N_c paired D4-branes located at $x=\phi_a$ and $x=-\phi_a$. On the other hand, the original curve (3.1) embeds naturally in the orientifold, and describes $N_c/2$ D4-branes at $x=\phi_a^2$. The overall factor of x indicates the presence of the orientifold plane at $x=0$.

The next thing to do is to try to understand some of the features of the string theory which might be immediately learned from this correspondence. First of all, the QCD scale Λ (which we have set to one in Eqs.(3.1) and (3.3)) characterizes the widths of the cuts in the z -plane. These cuts can be thought of as tubes or handles connecting the two sheets of the Riemann surface. At finite Λ , the embedding we have described should thus be modified by adding a compactified dimension to the D4-branes that run between the half-fivebranes. In this way, the solution of the $\mathcal{N}=2$ field theory reveals the internal

structure of D4-branes as objects of thickness Λ , which connect smoothly to the NS half-fivebranes according to the geometry of the Seiberg–Witten curve. At strong coupling, Λ becomes large (in an asymptotically free theory) and the internal structure becomes more apparent.¹⁰

If we are to identify all of the cuts and punctures of the z -plane with the locations of D4-branes ending on the NS half-fivebrane, we should also interpret the z^4 factor in the second term of the polynomial in (3.3). In the expression (3.1), written in terms of the ‘physical’ variables, where mirror points are removed, it is clear that *this point should be identified with two extra D4-branes*, which are forced to live on the orientifold, at $z = 0$. We should be careful though, as the introduction of two extra D4-branes should naively change the physics even away from strong coupling.

Recall that there is no non-trivial physics (associated with stable states) to be found in the $\mathcal{N}=2$ theory by examining monodromies around this point $z=0$. [27] Correspondingly, there should be no new physics arising in the weak coupling string theory either. This must mean that generically there are no new massless states coming from fundamental strings stretching from these branes to any other branes in the theory.

As far as the weakly coupled massless spectrum of the string theory is concerned, these two extra branes must remain completely invisible throughout our discussion of the previous section, until we come to the strong coupling regime. There, we anticipated that some extra branes appear in the theory which stay in the spectrum as we move to the magnetic theory. We had no means of fixing the number of such branes. *It is our conjecture that these ‘hidden’ D4-branes, apparent in the $\mathcal{N}=2$ theory’s polynomial, are exactly the two D4-branes which we sought in the previous section.* At the point where the NS–NS branes become coincident in the x^6 direction, these two branes appear in the theory on the same footing as all of the other branes, contributing to the massless spectrum as we move off to the magnetic theory.

We also expect that precisely the reverse must happen upon moving from the magnetic to the electric theory. This is perfectly consistent with the fact that neither NS–NS half-fivebrane can move off the orientifold and circle the other, a procedure that would result in an unacceptable increase in the number of new branes.

¹⁰ Ultimately, given that the strong coupling limit of the theory (which is locally type IIA) is supposed to be M-theory, we expect that the NS–NS fivebranes and D4-branes all become M5-branes in different configurations in an eleven dimensional theory. (The D4-branes unwrap a hidden leg wrapped around the hidden eleventh direction.) Alternatively, if we had first done a T_{23} -duality (the D4-branes becoming D2-branes and the D6-branes becoming D4-branes) and then gone to strong coupling, there would be an interesting M-theory configuration involving two M5-branes with stretched membranes between them. The strong coupling singularity of the string theory where they coincide is then identified with a point at which tensionless strings could arise.

Similar phenomena, of new states suddenly appearing in regions of moduli space, occur when one crosses so-called surfaces of marginal stability [13] in the moduli space of $\mathcal{N}=2$ gauge theories. As one crosses such surfaces (typically of codimension one), certain BPS states which were stable before crossing become unstable and *vice versa*. In the case at hand, we may suppose that the string states we might normally associate with the pair of D4-branes stuck on the orientifold are unstable to decay when the distance between the half-fivebranes is finite, so that their presence does not directly influence the spectrum. Such a state would consist of a string connecting a ‘hidden’ D4-brane at $z=0$ to a gauge D4-brane at $z=\phi_a$, together with its mirror image, running from $z=0$ to the mirror D4-brane at $z=-\phi_a$. We can imagine that this state would be unstable to decay to a state running directly between the other D4-brane and its mirror, which is already in the spectrum. Once the half-fivebranes cross and the extra D4-branes at $z=0$ become part of the magnetic gauge configuration, the string states would become stable gauge bosons.

This completes our justification for picking $\alpha=2$ for the $SO(N_c)$ theory in the previous section. As we can continuously move from our $\mathcal{N}=1$ configuration to the $\mathcal{N}=2$ situation by rotating a fivebrane[24], and from there move smoothly to the Coulomb phase where we see a sign of the two extra D4-branes, we expect that we should take their presence seriously, and anticipate that they might be relevant in the $\mathcal{N}=1$ theory. Admittedly, given the extra supersymmetry and the other special features of the rotated theory, we do not expect to be able to infer too much about the $\mathcal{N}=1$ theory this way, but we expect that at least the number of these hidden branes is preserved under the route we just described. In addition, we expect that given the similarities of the $USp(N_c)$ theory to the $SO(N_c)$ theory from the point of view of string theory, the *disappearance* of two D4-branes as one goes to the magnetic theory is also plausible.

4. Adjoint Matter

In this section, we briefly present our speculations (based on the conjecture of ref.[10]) on how we expect the inclusion of adjoint matter into our orientifolded models to work.

The models discussed so far are in fact closely related to models with a single matter field transforming in the adjoint of the gauge theory. As shown in [24], rotation of either the NS’ fivebrane or the NS fivebrane into the (x^4, x^5) or (x^8, x^9) directions preserves supersymmetry and when the two half-fivebranes lie in the same orientation, $\mathcal{N}=2$ supersymmetry is restored. The $N_c/2$ D4-branes are then free to move in the two of these directions that are shared by the world volumes of the fivebranes and correspond to the adjoint matter field’s vev. Thus, in the $\mathcal{N}=1$ configuration, the relative rotations of the fivebranes corresponds to a mass term for the adjoint fields.

As proposed in ref.[10], different superpotential terms for the adjoint may be included by the placement of extra NS fivebranes. In the SO/USp case the first non-trivial case is when

we include one extra *whole* NS–NS fivebrane¹¹, coincident with the NS half–fivebrane, but oriented like the NS′ brane. We expect that the resulting superpotential is $\text{tr}X^4$.

In general the addition of k extra coincident NS–NS fivebranes (oriented like the NS′ half–fivebrane) should generate the superpotential term $\text{tr}X^{2(k+1)}$. The dualities of [29] follow simply from this construction and our previous deductions. Each of the $2k+1$ half–fivebranes may be moved through the D6–branes, creating $N_f/2$ D4–brane connections to those D6–branes. Whole fivebranes may now freely be moved through the NS′ half–fivebrane carrying their N_f connections to the D6–branes. As these are whole fivebranes, we do not expect any new physics (over and above that found in ref.[10]) when we move them through. (In fact, as mentioned earlier, we expect that whole fivebranes can move off the O4–plane.)

Finally we move the NS half–fivebrane through the NS′ half–fivebranes with the resulting configurations discussed in the previous two sections. The final ‘magnetic’ configuration has the NS and NS′ fivebranes interchanged and connected by $kN_f + (N_f - N_c)/2 \pm 2$ D4–branes corresponding to the $SO/USp((2k+1)N_f - N_c \pm 4)$ dual gauge symmetry. The dual theory also possesses an adjoint field Y . There are $N_f(2k+1)/2$ D4–branes connecting the NS′ and N_f D6–branes. 4–4 strings in the final configuration supply N_f dual quarks.

The $N_f^2(2k+1)/2$ connections between D4 and D6–branes which are free to move in the (x^8, x^9) directions correspond to $(2k+1)$ mesons M_n in the $N_f(N_f+1)/2$ and the $N_f(N_f-1)/2$ representations of the flavour group. The resulting superpotential is of the form

$$\text{tr}Y^{2(k+1)} + \sum_{n=0}^{2k} M_n^{rs} q_r Y^{2k-n} \tilde{q}_s \quad (4.1)$$

where r and s are flavour indices.

5. Closing Remarks

This paper presents a framework in which the electric/magnetic dual descriptions of the physics of $\mathcal{N}=1$ supersymmetric $SO/USp(N_c)$ gauge theories with N_f quarks are embedded into string theory configurations. The idea is to realise the duality as a consequence of geometrical rearrangements of configurations of extended objects in string theory, in the spirit of refs.[1,10].

The addition of an orientifold plane allowed us to describe the $SO/USp(N_c)$ gauge theories as a modification of the presentation of ref.[10] for the $U(N_c)$ gauge theories. Although this is a simple modification to perform, it has very crucial consequences. It forced us

¹¹ Adding an odd number of half–fivebranes is probably not consistent. This would only flip the O4–plane charge an odd number of times.

go through a non-trivial situation (passing half-fivebranes through each other) in going between the magnetic and electric descriptions. Such a singular situation could be avoided in the case of $U(N_c)$, as shown in ref.[10], but not here.

We emphasize again that the necessity of going through such a singular configuration is not merely an inconvenience of the orientifold description. It is string theory's way of reproducing physics which is already anticipated from the point of view of field theory. In particular:

(i) Due to the absence of a $U(1)$ center for the SO/USp gauge groups, there are no allowed Fayet–Iliopoulos terms which may be included corresponding to the freedom to perform a movement which avoids the singularity.

(ii) There *must be* a phase transition in going between the electric and magnetic descriptions of $SO(N_c)$ gauge theories with quarks. That this is not the case for $U(N_c)$ is signaled by the possibility of avoiding the singular configuration. The only essential difference encountered between performing the rearrangement of branes for $U(N_c)$ and for the cases studied here is the necessity of the singular configuration and so we expect that this is related to the presence of the phase transition.

Due to the lack of a description of orientifolded NS–NS fivebranes, we were in an even worse position to describe the singular situation when they overlap than the analogous case for $U(N_c)$. However, we were able to smoothly deform the theory to an $\mathcal{N}=2$ model in its Coulomb phase, where we were able to check some of our assumptions about the new features which must arise at point where the NS–NS fivebranes are coincident. To do so, we were able to identify the auxiliary higher genus (Seiberg–Witten) surface associated with the vacua of the $\mathcal{N}=2$ theory with the configurations of fivebranes and D4–branes which was present. This identification allowed us to identify a pair of extra D4–branes which must appear in the theory as we go between phases¹².

In closing, we note that there are many avenues of investigation to pursue further. Chief among these is the issue of how much more about the physics of extended objects (and the resulting field theories they encode) can one learn by studying the powerful results of $\mathcal{N}=2$ field theory. In this paper, we have found a correspondence between the Seiberg–Witten curve and configurations of D4–branes ending on NS–NS fivebranes. There are undoubtedly many more entries to be put into the dictionary which translates between the physics of extended objects and the physics of exactly solvable field theories, which all will be of great value in continuing to understand duality in both field theory and string theory.

¹² One can also carry out this procedure in the $U(N_c)$ case, where, upon examination of the Seiberg–Witten curve, it is clear that there are no extra D4–branes.

Acknowledgments:

We would like to thank Philip Argyres and Michael Crescimanno for discussions, and Eric Gimon for helpful comments on the manuscript. The work of NE was supported in part by the Department of Energy under contract #DE-FG02-91ER40676. ADS's research was supported in part by DOE EPSCoR grant #DE-FC02-91ER75661 and by an Alfred P. Sloan Fellowship. NE would also like to thank the Department of Physics and Astronomy at the University of Kentucky for hospitality while some of this research was carried out. CVJ would like to thank the Physics Department at Stanford University for their kind hospitality and enjoyable working environment while this research was completed.

References

- [1] A. Hanany and E. Witten, hep-th/9611230.
- [2] K. Intriligator and N. Seiberg, Phys. Lett. **B387** (1996) 513; hep-th/9607207.
- [3] A. Strominger, Nucl. Phys. **B343**, (1990) 167, *erratum* **353** (1991) 565.
- [4] J. Dai, R. G. Leigh and J. Polchinski, Mod. Phys. Lett. **A4** (1989) 2073;
P. Hořava, Phys. Lett. **B231** (1989) 251;
R. G. Leigh, Mod. Phys. Lett. **A4** (1989) 2767;
J. Polchinski, Phys. Rev. D50 (1994) 6041, hep-th/9407031.
- [5] A. Sagnotti, in ‘*Non-Perturbative Quantum Field Theory*’, Eds. G. Mack *et. al.* (Pergammon Press, 1988), p521;
V. Periwal, unpublished;
J. Govaerts, Phys. Lett. **B220** (1989) 77;
P. Horava, Nucl. Phys. **B327** (1989) 461.
- [6] J. Polchinski, hep-th/9611050.
- [7] M. J. Duff, J. X. Lu and R. R. Khuri, Phys. Rept. **259** (1995) 213; hep-th/9412184.
- [8] N. Seiberg, Nucl. Phys. **B435**, (1995) 129; hep-th/9411149.
- [9] K. Intriligator and N. Seiberg, Nucl. Phys. **B444** (1995) 125; hep-th/9503179.
- [10] S. Elitzur, A. Giveon and D. Kutasov, hep-th/9702014.
- [11] H. Ooguri and C. Vafa, hep-th/9702180.
- [12] T. Banks and E. Rabinovici, Nucl. Phys. **B160** (1979) 349;
E. Fradkin and S. Shenker, Phys. Rev. **D19** (1979) 3682.
- [13] N. Seiberg and E. Witten, Nucl. Phys. **B426** (1994) 19, hep-th/9407087; Nucl. Phys. **B431** (1994) 484, hep-th/9408099.
- [14] E. Witten, hep-th/9703166.
- [15] M. R. Douglas and G. Moore, hep-th/9603167.
- [16] J. Polchinski, hep-th/9606165, to appear in Phys. Rev. **D**.
- [17] C. V. Johnson and R. C. Myers, hep-th/9610140, to appear in Phys. Rev. **D**.
- [18] E. G. Gimon and J. Polchinski, Phys. Rev. **D54** (1996) 1667, hep-th/9601038.
- [19] P.C. Argyres, M.R. Plesser, and A.D. Shapere, Nucl. Phys. **B483** (1997) 172, hep-th/9608129.

- [20] C. G. Callan, J.A. Harvey and A. Strominger, Nucl. Phys. **B359** (1991), 611; Nucl. Phys. **367** (1991), 60.
- [21] P. Horava, Phys. Lett. **B289** (1991) 293; hep-th/9203031.
- [22] L. Dixon, J. Harvey, C. Vafa and E. Witten, Nucl. Phys. **B261** (1985) 678; Nucl. Phys. **B274** (1986) 285.
- [23] E. G. Gimon and C. V. Johnson, Nucl. Phys. **B477** (1996) 715, hep-th/9604129.
- [24] J. Barbón, hep-th/9703051.
- [25] P.C. Argyres, M.R. Plesser, and N. Seiberg, Nucl. Phys. **B471** (1996) 159, hep-th/9603042.
- [26] P. Argyres and A. Shapere, Nucl. Phys. **B461** (1996) 437, hep-th/9509175.
- [27] A. Brandhuber and K. Landsteiner, Phys. Lett. **B358** (1995) 73, hep-th/9507008.
- [28] U.H. Danielsson and B. Sundborg, Phys. Lett. **B358** (1995) 273, hep-th/9504102.
- [29] R. G. Leigh and M. Strassler, Nucl. Phys. **B356** (1995) 492; hep-th/9505088.